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Magnetic Field Propagation in a Stellar Dynamo

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Numerical simulations of stellar dynamos are reviewed. Dynamic dynamo models solve the nonlinear, three-dimensional, time-dependent, magnetohydrodynamic equations for the convective velocity, the thermodynamic variables, and the generated magnetic field in a rotating, spherical shell of ionized gas. When the dynamo operates in the convection zone, the simulated magnetic fields propagate away from the equator in the opposite direction inferred from the solar butterfly diagram. When simulated at the base of the convection zone, the fields propagate in the right direction at roughly the right speed. However, owing to the numerical difficulty, a full magnetic cycle has not been simulated in this region. As a result, it is still uncertain where and how the solar dynamo operates.

KEY WORDS: Numerical simulations; stellar dynamos.

1. INTRODUCTION

The understanding of the basic mechanisms of a stellar dynamo, especially the solar dynamo, has been the object of considerable effort in recent years owing to the lack of a self-consistent explanation of the solar butterfly diagram. This diagram is a record of where, in solar latitude, sunspots have been observed as a function of time. It clearly depicts the 11-year period in the total sunspot area and the equatorward drift of the active solar latitude during each eleven years. The Sun's 22-year magnetic cycle[(1)] was discovered by observing the polarity configuration of sunspot groups. Buoyant magnetic flux tubes presumably break away from a main toroidal magnetic field below the surface and emerge as sunspots. Duing the first half of the magnetic cycle, the polarity configuration of the leading and following sunspots is the same for each group in the northern hemisphere

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while the sunspot groups in the southern hemisphere have the opposite configuration; the pattern is reversed during the second half of the cycle. To satisfy this polarity law, two oppositely directed toroidal fields, one in each hemisphere, are assumed to propagate from midlatitude to the equator in eleven years followed by a similar scenario during the next eleven years with the opposite polarity.[2,3]

Parker[4] proposed an explanation for the maintenance of a cyclic stellar magnetic field. He assumed that the radial and latitudinal variation in the rotation rate, i.e., the differential rotation, generates a toroidal, i.e., longitudinal, magnetic field by shearing the poloidal, i.e., meridional, magnetic field and that helical convective motions regenerate a reversed poloidal field by twisting the toroidal field. In addition, the magnetic fields are assumed to be continuously diffusing away via magnetic eddy diffusion. This explanation predicts a field propagation toward the equator if angular velocity increases with depth and left-handed (right-handed) fluid motions dominate in the northern (southern) hemisphere; or vice versa. Also, the larger the helicity and differential rotation, the shorter the period of the magnetic cycle. In addition, it must be assumed that the effect of helicity is small relative to differential rotation because helicity can also generate a toroidal magnetic field by twisting the poloidal field, and this effect usually inhibits magnetic field propagation.

Dynamo models have been developed to test this hypothesis. The vast majority of these models have been kinematic models[5–9] which obtain an axisymmetric magnetic field from a linear magnetic induction equation that is tuned by independently parametrizing the effects of differential rotation and helicity. These models have been relatively successful in simulating a dynamo wave that propagates toward the equator with about the right period. Usually helicity is assumed to be left-handed in the northern hemisphere and right-handed in the southern hemisphere because the Coriolis forces resulting from the expansion (contraction) of rising (sinking) gas in a stratified, rotating convection zone will produce such a helicity profile. Consequently, in order to get the magnetic fields to propagate toward the equator, angular velocity is assumed to increase with depth through the convection zone.

Another approach to modeling a stellar dynamo has been to solve a set of truncated, nonlinear dynamo equations. Although much of the physics is removed in such a model, enough is retained to study the nonlinear effects of the feedbacks between the velocity and magnetic fields that are responsible for the variation in cycle period and amplitude. This approach is reviewed by N. O. Weiss in this issue.

Dynamic dynamo models simultaneously and self-consistently solve the magnetohydrodynamic equations for the velocity, thermodynamic variables, and the magnetic field with full nonlinear feedback, in three dimensions and time, explicitly including the effects of spherical geometry and rotation. Dynamic models for a Boussinesq fluid,[10,11] having no basic density stratification, and for an anelastic gas,[12,13] having a significant density stratification, have numerically simulated dynamos in rotating convection zones; however, the simulated magnetic fields propagate away from the equator (contrary to what is inferred from the solar butterfly diagram) with a period shorter than 22 years. The reason for this is that, although the simulated helicity in dynamic models is left-handed (right-handed) in the northern (southern) hemisphere as assumed in the kinematic models, the simulated angular velocity decreases with depth. In addition, since the simulated helicity in the convection zone is larger than assumed in kinematic models, the simulated magnetic fields propagate too fast.

Those who model kinematic dynamos have argued that the differential rotation profile numerically simulated in dynamic models must not be correct. However, the latitudinal variation in angular velocity at the surface is in agreement with Doppler measurements of the solar surface rotation rate[14] and the radial variation is in agreement with a recent analysis[15] of the rotational frequency splitting of solar oscillations.[16] Physical explanations, based on the analysis of three-dimensional numerical simulations,[10–13] have been made for the maintenance of these profiles.

In an attempt to resolve this problem and others concerning the real possibility that the solar convection zone is too turbulent to maintain large-scale magnetic fields, it has been proposed that the solar dynamo may be operating at the base of the convection zone in the transition region between the stable interior and the turbulent convective region.[17–19] Dynamic dynamo simulations at the base of the convection zone[20] support this hypothesis; however, owing to the limited numerical resolution of the small amplitude motions in the stable region, a full magnetic cycle was not simulated.

I will review some of the dynamic dynamo simulations. First I will describe those with the dynamo operating in the convection zone; then those with the dynamo operating at the base of the convection zone.

2. IN THE CONVECTION ZONE

Gilman's[10,11] pioneering work on dynamically consistent dynamo simulations for a rotating, spherical fluid shell governed by the three-dimensional, nonlinear, magnetohydrodynamic equations has demonstrated that the mechanisms suggested by Parker[4] can maintain a self-excited cyclic dynamo. A magnetic field is generated by the shearing and transport mechanisms of the differential rotation and convective motions. It grows until the Lorentz forces become large enough to alter the fluid motions that maintain the field. This nonlinear feedback produces time-dependent kinetic and magnetic energies. Although beautiful simulations were obtained over several cycles, the magnetic fields propagate away from the equator in the opposite direction inferred from the solar butterfly diagram with a period about an order of magnitude shorter than the solar period.

I developed a dynamic dynamo model[12] that differs from Gilman's in several ways. The numerical technique is based on a spectral representation with a semi-implicit time-integration scheme, whereas Gilman's model is based on a finite difference representation with an explicit time-integration scheme. The major difference is that I model an anelastic fluid which accounts for the effects of a large density stratification. (Gilman has since converted his Boussinesq model to an anelastic model.) However, my anelastic dynamo model also produces magnetic fields that propagate away

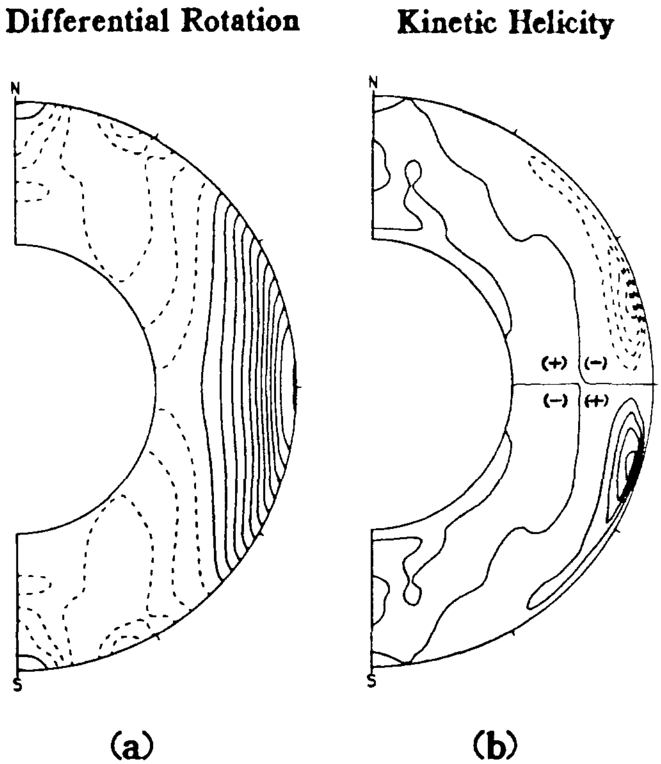

Fig. 1. From Glatzmaier.[13] (a) Solid (broken) contours represent positive (negative) angular velocity relative to the rotating frame of reference. (b) Solid (broken) contours represent positive (negative) helicity averaged in longitude.

from the equator.[13] Although I have not simulated a complete magnetic cycle, the fields appear to propagate at about twice the speed observed on the sun.

The simulated differential rotation and helicity profiles are illustrated in Fig. 1 for the anelastic model and in Fig. 21 of Ref. 10 for the Boussinesq model. As indicated in both figures, angular velocity increases with distance from the rotation axis, in agreement with what is inferred from the frequency splitting of solar oscillations.[15,16] At the surface, angular velocity increases toward the equator in agreement with Doppler measurements of the solar surface rotation rate.[14] These figures also illustrate how helicity, which is the dot product of velocity and its curl, tends to be negative, left-handed in the northern hemisphere and positive, right-handed in the southern hemisphere. Helicity in Fig. 1 peaks near the surface because, due to mass conservation, fluid velocity is large where the mass density is small. These profiles of differential rotation and helicity have evolved because of the effects of rotation, spherical geometry, and density stratification.[10–13]

Now consider how these velocity profiles maintain propagating magnetic fields. Typical axisymmetric parts of the toroidal and poloidal magnetic fields are illustrated in Fig. 2 and in Fig. 10 of Ref. 11. Toroidal

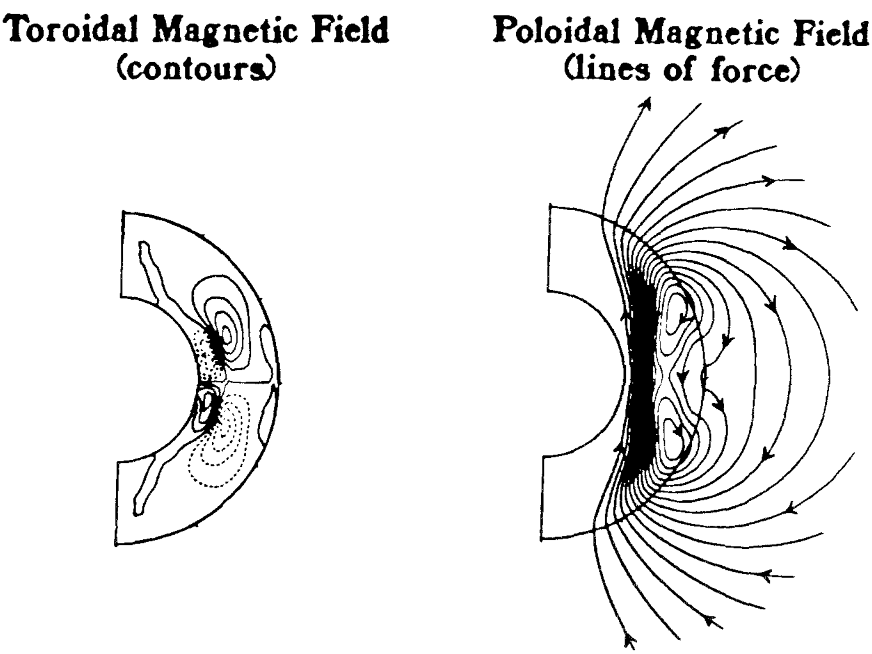

Fig. 2. From Glatzmaier.[13] Solid (broken) contours represent the toroidal magnetic field into (out of) the paper. Lines of force represent the poloidal magnetic field.

fields are generated by rotationally shearing the poloidal fields. As discussed by Gilman[11] and Glatzmaier,[13] this tends to enhance the toroidal fields on their poleward sides and destroy them on their equatorward sides. As a result, the toroidal field pattern propagates away from the equator. The poloidal field is regenerated with the opposite polarity by the helical fluid motions which twist the toroidal field. The resulting phase propagation of the simulated magnetic field is illustrated in Fig. 3 of Ref. 13 and in Figs. 10 and 11 of Ref. 11. However, as already mentioned, these fields are propagating in the opposite direction inferred from the solar butterfly diagram.

3. AT THE BASE OF THE CONVECTION ZONE

In an attempt to resolve this problem, I modeled a dynamic dynamo in the convective overshooting region, i.e., the inner half of the spherical shell, by numerically decoupling the velocity and magnetic fields in the outer (superadiabatic) half of the shell.[20] If the solar convection zone is turbulent enough to concentrate the majority of the magnetic flux into thin tubes as observed on the solar surface,[21] the velocity and magnetic fields will be spatially separated and physically decoupled to some extent.[19]

Again, the two major factors responsible for magnetic field propagation are differential rotation and helicity. Angular velocity in the inner part of the shell decreases with depth as it does in the outer part; however, helicity tends to have the opposite sign in the inner part of the shell (Fig. 1). Fluid tends to converge while sinking through the stratified-convection zone but diverges slightly as it terminates its descent near the base of the convective cells, and vice versa for rising fluid. Consequently, the Coriolis forces cause the fluid to twist in one direction as it sinks through the outer part of the shell and in the opposite direction as it sinks through the inner part. As explained by Glatzmaier,[20] the simulated magnetic fields at the base of the convection zone are enhanced on their equatorward sides and destroyed on their poleward sides; consequently, they propagate toward the equator. In addition, due to the small amplitude of the simulated helicity in this region, the fields initially propagated at roughly the right phase speed. However, due to the limited numerical resolution of the small amplitude helical fluid motions in the stable inner part of the shell, only a fraction of a magnetic cycle could be simulated.

4. SUMMARY

I have briefly reviewed three-dimensional, nonlinear simulations of dynamic dynamos operating in the convection zone, where the simulated magnetic fields propagate too fast and in the wrong direction, and at the base of the convection zone where the initial propagation is in the right direction and at roughly the right speed. Although these simulations suggest that the solar dynamo may be seated at the base of the convection zone, it is uncertain how Hale's polarity law[(1)] for sunspot groups observed on the solar surface could be maintained so well by flux tubes originating at the base of the convection zone. On the other hand, the dynamo may be operating in the outer 5% of the sun (by radius), where angular velocity appears to be increasing with depth.[(15,22)] We have been unable to study this hypothesis with our global models owing to the large numerical resolution requirements in this rapidly varying, thin, outer layer. However, if the dynamo were operating in this top layer with concentrated magnetic flux tubes, the mechanism probably would be quite different from what is represented by current dynamo theory.

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